Energy Games for Generalized Equivalence Checking

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Abstract

We discuss how to solve whole *hierarchies of behavioral equivalence problems* as single *quantitative problems* about *modal logics* through *energy games*.

Deciding All Equivalences at Once Research in program semantics has come up with many notions of behavioral equivalence for concurrent programs. Famous notions are trace equivalence, similarity, bisimilarity, failure trace equivalence, and their *weak* versions that abstract over internal behavior. Sometimes, one wonders which equivalences (or preorders) apply to a specific pair of systems. While there are algorithms to decide individual equivalences, trying out various checkers is cumbersome and prone to errors. Would it not be nice to *decide all behavioral equivalences at once*?

My PhD thesis on *generalized equivalence checking of concurrent programs* [Bis25] proposes the first unified way to check for whole ranges of equivalences. Its prototype implementation https://equiv.io can compute which notions from the strong and weak linear-time–branching-time spectrum [vG90, vG93] relate pairs of finite-state processes (see Figure 1).

The Energy Game Approach The approach (Figure 2) relies on understanding equivalence spectra through hierarchies of Hennessy–Milner sublogics (HML). The game-based algorithms then compute "how big a logic" one needs to tell two states apart. Smaller logics correspond to weaker equivalences. We dub the problem of finding smallest such logics the *spectroscopy problem*.

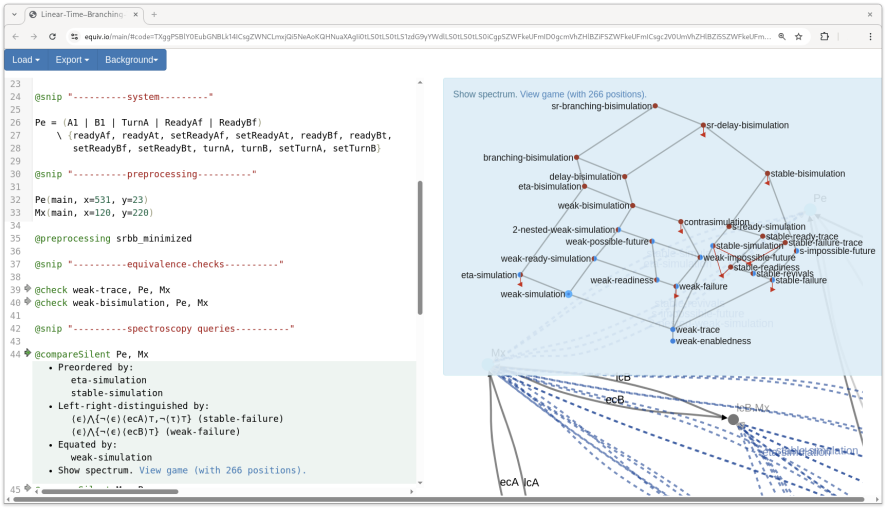

Figure 1: https://equiv.io naming all weak preorders between a CCS implementation and specification of Peterson's mutual exclusion protocol.

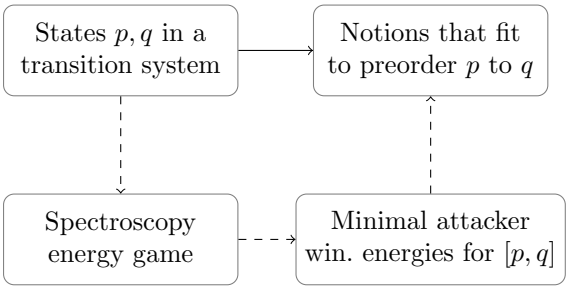

Figure 2: Approach to solve the spectroscopy problem (solid line) through games (dashed lines).

Figure 3: D-CON'25 audience examining a 3-dimensional energy bisimulation game.

We solve the spectroscopy problems of specific equivalence spectra through *spectroscopy games*. These are *energy games* generalizing the classic bisimulation game. In such games, the energies bound the possibilities of an attacker trying to tell states apart. Game moves and productions in modal languages align such that attacker's winning energies reveal modal languages that contain distinguishing formulas and, dually, maintained preorders.

Strictly speaking, the games employed are no classical energy games but a generalization, which we call *Galois energy games* [LB25]. The idea to abstract the bisimulation game to capture all classic equivalences [vG90] through their modal logics originates in [BJN22]. With [Bis23], the approach is simplified to use (Galois) energy games. In [BJ24, BHL+25], we use it to address weak equivalences [vG93].

The Talk My talk is meant to present the core ideas behind my thesis and the underlying publications: How to solve whole *hierarchies of qualitative equivalence problems* as single *quantitative problems* about *modal logics* through *energy games*. As example, we will discuss how to tackle the spectroscopy problem for the P-easy part of equivalence problems by adding 3-dimensional energies to the bisimulation game (3-d for real, see Figure 3!).

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